# 等分散性の検定に対する二つの方法が等価でない場合

# A Case in which Two Methods for the Test of Homogeneity of Variance are not Equal

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Regarding the test of homogeneity of variance based on the F-distribution, there are two methods for accepting or rejecting the null or alternative hypothesis. In this paper, it is shown that there is a case in which the two methods are not equal if the level of significance $\alpha$ is an unordinary value ($\alpha = 0.98$). In ordinary cases of statistical processing, the value of $\alpha$ is 0.05 or 0.01. The two methods are equal in such cases.

F-分布に基づく等分散性の検定について、帰無仮説と対立仮説の採択および棄却に関し、二つの方法がある。本稿では、有意水準 $\alpha$ が極端な値 ($\alpha = 0.98$) である場合、これら二つの方法は等価ではなくなることを示す。なお、統計処理における通常の場合、$\alpha$ の値は $\alpha = 0.05$ や 0.01 であるのが慣例であり、この範囲で、二つの方法は等しい。

Keywords:
| | |
|---|---|
| 分散 | variance |
| 等分散 | homogeneity of variance |
| 仮説検定 | statistical hypothesis testing |
| F-分布 | F-distribution |
| 等分散の検定 | test of homogeneity of variance |

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# I. Introduction

In this paper, it is shown that there are two methods for accepting and rejecting the hypothesis in the test of homogeneity of variance, and it is shown that there is a case in which those two methods are not equivalent.

The meaning of each letter, such as $F, \alpha$, $f_1$ and $f_2$ is given the first time it appears in this paper, and the letters are used thereafter without an explanation of their meaning.

There are arguments[4] [5] about accepting the null hypothesis when it is not rejected. The author does not go into detail about the arguments and takes boolean expressions that have only two values, $\mathsf{T}$ and $\neg\mathsf{T} = \mathsf{F}$.

# II. Test of Homogeneity of Variance based on *F*-distribution

We assume that two samples, $A$ and $B$, both have the Normal distribution.

Let variances of samples $A$ and $B$ be $\sigma_A^2$ and $\sigma_B^2$, respectively. The null hypothesis $H_0$ and the alternative hypothesis $H_1$ are shown below:

$$\begin{cases} H_0: & \sigma_A^2 = \sigma_B^2 \\ H_1: & \sigma_A^2 \neq \sigma_B^2 \end{cases}$$

Let values that are calculated by the point estimates of the two variances $\sigma_A^2$ and $\sigma_B^2$ be $\hat{\sigma}_A^2$ and $\hat{\sigma}_B^2$, respectively. The ratio $F = \hat{\sigma}_A^2/\hat{\sigma}_B^2$ is a test statistic in the test of homogeneity of variances. It follows an F-distribution[7] [8] under the condition in which the null hypothesis $H_0$ is confirmed.

The F-distribution is characterized by two degrees of freedom $d_A$ and $d_B$. Let the size of a sample $x$; $(x = A, B)$ be $N_x$, respectively. Then the degree of freedom is $d_x = N_x - 1$.

As the F-distribution, values $f_1$ and $f_2$[*1] set two-tailed critical regions $[0, f_1)$ and $(f_2, \infty)$ for the alternative hypothesis $H_1$, and the values are calculated by the expressions shown below (∗).

$$\begin{cases} f_1 & = \mathrm{qf}\left(1 - \dfrac{\alpha}{2}, d_A, d_B\right), \\ f_2 & = \mathrm{qf}\left(\dfrac{\alpha}{2}, d_A, d_B\right). \end{cases} \quad \ldots (*)$$

The function qf shown above is defined as

$$\mathrm{qf}(a, n_1, n_2) = F_{n_1,n_2}(a).$$

According to an explanation in a reference[2](p. 65), a value $t$ for which $P(F \geq t) = a$ holds is written as $F_{n_1,n_2}(a)$.

# III. Two Different Methods

In this section, we show two methods for accepting and rejecting the null hypothesis.

## 1. Method One (M1)

In a book[1](p. 143), the conditions for accepting and rejecting the null hypothesis are explained as follows:

$$\begin{cases} f_1 \leqq F \leqq f_2 & \Longleftrightarrow \quad H_0, \\ F < f_1 \text{ or } f_2 < F & \Longleftrightarrow \quad H_1. \end{cases}$$

Each $H_i$ $(i = 0, 1)$ stands for accepting the hypothesis $H_i$ on the right-hand side of the equivalent symbol $\Longleftrightarrow$ .

*1 Those are called 'percentile' or 'quantile.'

## 2. Method Two (M2)

On the other hand, there is another method in a book[3](p. 313).

$$\begin{cases} 1 \leqq F \text{ then } \quad f_2 < F & \Longleftrightarrow H_1, \\ F < 1 \text{ then } \quad f_1' < \dfrac{1}{F} & \Longleftrightarrow H_1, \\ \qquad \text{otherwise} & \Longleftrightarrow H_0. \end{cases}$$

The percentile $f_1'$ in the second row is defined by a function qf as shown below:

$$f_1' = \mathrm{qf}\left(\frac{\alpha}{2}, d_B, d_A\right).$$

The order by which degrees of freedom $d_B$ and $d_A$ appear in the expression of the definition for $f_1'$ is reverted against $f_1$ and $f_2$. The relationship*2 between $f_1'$ and $f_1$ is shown below:

$$\begin{aligned} f_1' &= \mathrm{qf}\left(\frac{\alpha}{2}, d_B, d_A\right) \\ &= \frac{1}{\mathrm{qf}\left(1 - \dfrac{\alpha}{2}, d_A, d_B\right)} = \frac{1}{f_1} \qquad \cdots (\dagger) \end{aligned}$$

# IV. Discussion

We show in this section that there is a case in which the two methods (M1 and M2) shown above are not equivalent.

## 1. M1 $\Longrightarrow$ M2

Assume that $f_1 \leqq F \leqq f_2$ in M1. In the case of $1 \leqq F$, we accept $H_0$ in M2 because it holds that $F \leqq f_2$ and then this is the case of 'otherwise' in M2. In the case of $F < 1$, it holds that $f_1 \leqq F$ and the relationship (†) implies that $1/F \leqq 1/f_1 = f_1'$ holds. This is the case of 'otherwise,' and then we accept $H_0$ in M2.

Therefore, it always holds that $H_0$ in M1 $\Longrightarrow$ $H_0$ in M2.

## 2. M2 $\Longrightarrow$ M1

Assume that $1 \leqq F$ and $F \leqq f_2$. This is a case of accepting $H_0$ in M2. For example, let $\alpha = 0.98$, $d_A = 30$ and $d_B = 10$. Then, by expressions (∗) we have $f_2 = \mathrm{qf}(\alpha/2, d_A, d_B) = 1.061$ and $f_1 = \mathrm{qf}(1 - \alpha/2, d_A, d_B) = 1.033$. These values are calculated by **R***3*4(`lower.tail = F`). $F$ does not depend on $\alpha$, $d_A$ and $d_B$, and $F$ takes any value for which $1 \leqq F \leqq f_2$ holds. For example, let $F = 1.024$, then it holds that $1 \leqq F < f_1 < f_2$ (Fig. 1). This is a case of accepting $H_1$ in M1.

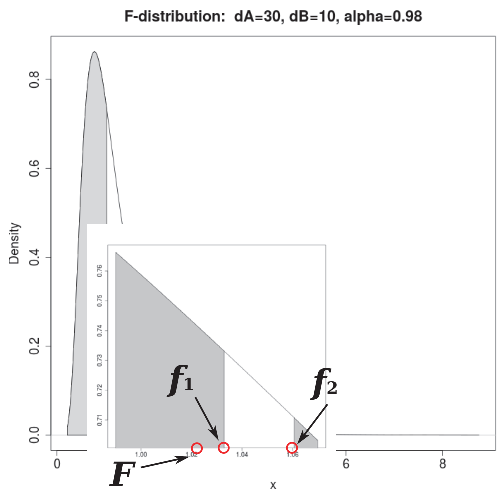


Figure1　A case of accepting $H_1$ in M1

Therefore, there is a case in which it does not hold that $H_0$ in M2 $\Longrightarrow$ $H_0$ in M1.

## 3. $\alpha = 0.98$

In ordinary cases, the value of the level of significance $\alpha$ is $\alpha = 0.05$ or $\alpha = 0.01$. The two methods

*2 It is proven in **http://www.econ.hokudai.ac.-jp/~takagi/2005_July_2nd.pdf**, and so on.

*3 version 3.2.3

*4 **http://www.R-project.org/**

are equal in such ordinary cases[6].

In the case shown above, we let $\alpha$ be 0.98. In ordinary cases of statistical processing, such a value $\alpha = 0.98$ is never used. The case of $\alpha = 0.98$ is unordinary.

The level of significance $\alpha$ stands for probability. In a general mathematical meaning however, $\alpha$ can have any value in the range $0 \leqq \alpha \leqq 1$, then the case of $\alpha = 0.98$ is possible.

## V. Conclusion

This paper shows that there are two methods for the test of homogeneity of variance based on F-distribution and shows that there is a case in which they are not equivalent.

In ordinary cases, the value of the level of significance $\alpha$ is $\alpha = 0.05$ or $\alpha = 0.01$. In the case shown above, we let $\alpha$ be 0.98 and it is possible but unordinary.

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## Notes

This paper is a revised version of a paper[6].

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It is not the author's intention to consider the authors of books[1] [3] bad or wrong. The author has respect for them as great statisticians and thanks them for providing an opportunity to study statistics.